US Open Mathematical Finance Journal
Vol. 1, No. 1, January 2014, PP:1-15
Available online at http://arepub.com/Journals.php

Research article

Reflecting the Influence of Taxation of Income in Discounted Cash Flow Models

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Abstract

This article views the procedures of adjusting the expected cash flows on the before-tax and after-tax calculation bases; shows the conditions for the convergence of calculation results and gives the adjusted formulas for the models of infinite growth; and specifies the type of the cost of capital for equity capital, used for calculating a *weighted average cost of capital* (WACC).

Keywords: pretax discount rate, after-tax discount rate, valuation on the before-tax basis, valuation on the after-tax basis.

Introduction

It is known that the discounted cash flow methods/models and those of capitalized cash flows and earnings are the two most popular types of income approach methods as soon as income-bearing items (those generating positive cash flows) are evaluated. The theory of corporate finance [1,2,3] clearly states two fundamental principles being basic for discounting.

The 1st principle – the ruble got/paid tomorrow costs cheaper than that got/paid today.

The 2nd principle – a safe ruble is more expensive than a risky one; i.e., the expectations related to earning income from safe projects or income-bearing ones are estimated higher than those related to riskier projects or less income-bearing assets.

One more principle, a 3rd principle, which is basic for the models of discounted cash flows and those of capitalized cash flows and earnings, says that valuating income-bearing items must strictly observe the relationship between the "numerator and denominator"; i.e., for each type of cash flows (in the numerator), a certain discount or capitalization rate is to be applied (in the denominator).

All the aforementioned basics are widely known. The author of the paper perfectly realizes that the general culture of applying both the models of discounting and those of capitalization has been significantly improved in the recent years; nowadays, one can rarely meet valuation reports, in which, for the cash flows nominated in RUB, the estimator uses real discount rates intended for the flows nominated in USD. In other words, the majority of estimators and investment analysts know the principle of "the relationship between the numerator and denominator", in accordance to which:

- To adjust nominal future cash flows for a certain valuation date, nominal discount rates are to be used (both positions include inflation);
- To adjust real future cash flows for a certain valuation date, real discount rates are to be used (both positions exclude inflation);
- To adjust future cash flows nominated in a certain currency for a certain valuation date, the discount rates nominated in the same currency are to be used;
- To adjust risk-free cash flows for a certain valuation date, risk-free discount rates are to be used;
- To adjust risky (uncertain) cash flows for a certain valuation date, the discount rates found with regard to risks are to be used;
- To adjust equity cash flows for a certain valuation date, yield rates required by shareholders for participating in the company's equity assets are to be used;
- To adjust cash flows from the entire invested assets for a certain valuation date, weighted average yield rates required by all the investors (shareholders and debtors);
- To adjust before-tax cash flows for a certain valuation date, before-tax alternative yield rates (i.e., before-tax discount rates) are to be used;
- To adjust after-tax cash flows for the valuation date, after-tax alternative yield rates (i.e., after-tax discount rates) are to be used;

Most of the aforementioned types of relationship regular for the principle of "the relation between the numerator and denominator" are both widely popular and, moreover, easy for their practical application. Nevertheless, there are several exceptions in this list – here we speak about the two last types of relationship given hereinabove, which account for the tax factor, i.e., the influence of the taxation of both income and dividends on the discount rate. The material given further hereon reflects the author's position on specifying the correct ways to practically use the principle of "the relationship between the numerator and denominator" either with regard to or regardless of the aforementioned taxes. The objective of the research[1] is to emphasize the correctly discounted cash flows of expected earnings in prevention of mixing the various types of both cash flows and applied discount and capitalization rates.

Let's start with the models of discounted/capitalized cash flows to equity and proceed with the more complex models of discounted/capitalized flows to the invested capital.

1. Models of discounted/capitalized cash flows to equity

Let's view the formation and calculation of cash flows for a certain period *on the level of the equity issuer*:

$$FCFE = NI + D -/+ Inv +/- New\ Debt\ issue = \qquad (1)$$

$$= NI \times (1 - k) = EBT(1-t)(1-k), \qquad (2)$$

Where:

FCFE – The balance of the free cash flows to the equity issuer, i.e., the balance of cash flows to shareholders after the income tax withheld and before the personal dividend tax levied;

NI – The net income of shareholders;

D – The depreciation;

Inv – The summed investments (-) and disinvestments (+) in non-current assets and current capital for a certain period;

1 Being an integral part of the paper [4]

New Debt issue – The newly attracted loans at interest minus the returned loan funds for a certain period;

k – The share of net reinvestments from net income;

EBT – The earnings before taxes;

t - The income tax rate.

Let’s view the formation and calculation of cash flows for a certain period *on the level of shareholders:*

$$FCFE_S = FCFE\,(1 - t_d) = \tag{3}$$

$$= div_н(1- t_d) = div_в, \tag{4}$$

Where:

$FCFE_S$ - The balance of free cash flows to equity for a certain period on the level of shareholders;

t_d - The dividend tax rate (e.g., in Russia this rate amounts to 9% for shareholders-natural persons being residents; 15% for shareholders-legal entities being residents; 30% for shareholders-natural persons being nonresidents; 20% for shareholders-legal entities being nonresidents);

$div_н$ – The amount of dividends accrued for a certain period;

$div_в$ – The sum of dividends received for a certain period.

It is necessary to point out that equation (3) doesn’t take into account the possibility of a so-called partial payment of dividend (i.e., the accumulation of cash reserve); and equation (4) neglects both the possibility of shareholders’ contribution to the authorized capital and that of the outside realization of holders’ shares (although, in general, including equation (3), it is, undoubtedly, possible). Contributions to equity capital aren’t taken into account as these fees are left in the past for current shareholders, and the income approach method uses only the events expected in the future.

Applying the models of capitalization, the following equations are obviously to be satisfied:

$$V = \frac{FCFE_s}{r_s - g} = \frac{FCFE(1-t_{def})}{r_s - g} = \frac{FCFE_{BT}(1-t_{efE})}{r-g} = \frac{FCFE}{r-g} = \frac{FCFE_{BT}}{r_{BT}-g}, \tag{5}$$

Where:

V – The cost of the income-bearing item subject to valuation;

$FCFE_S$ – The after-tax free cash flow to equity expected in the nearest period on the level of shareholders;

FCFE – The after-tax free cash flow to equity expected in the nearest period on the level of the equity issuer;

$FCFE_{BT}$ – The before-tax free cash flow to the equity issuer, expected in the nearest period;

t_{def} - The effective dividend tax rate (the share of “dividend” tax in the iafter-tax cash flow to the equity issuer (those after income tax withheld));

t_{efE} – The effective income tax (see the data hereinafter);

r_s – The after-tax rate of the discounted cash flow to equity;

r – The after-tax rate of the issuer’s discounted cash flow;

r_{BT} – The before-tax rate of the issuer’s discounted cash flows;

g – The expected rate of growth in cash flow on the before-tax and after-tax bases.

Here we make allowance for the fact that in a long-term period the rate of growth in cash flow is both similar and permanent on the level of the issuer and on that of shareholders (moreover, on both the before-tax and after-tax bases).

From equation (5), the following relationship between r and r_{BT} becomes obvious:

$$r = g + (r_{BT} - g)(1 - t_{efE}) = r_{BT}(1 - t_{efE}) + g t_{efE}, \tag{6}$$

$$r_{BT} = g + \frac{r - g}{1 - t_{efE}} = \frac{r - g t_{efE}}{1 - t_{efE}}, \qquad (7)$$

Equations (6) and (7) are analogue to the following equations connecting the values of before-tax and after-tax discount rates for cash flows on the level of shareholders:

$$r_s = g + (r_{BT} - g)(1 - t_{efE})(1 - t_{def}), \qquad (8)$$

$$r_{BT} = g + \frac{r_s - g}{(1 - t_{efE})(1 - t_{def})}, \qquad (9)$$

From equations (8) and (9), the parameter t_{efE} - effective income tax rate – is suggested to be calculated as:

$$t_{efE} = \frac{t}{1-(1-t)k} = \frac{0.2}{1-0.8k} = \frac{EBT \times t}{EBT + A - Inv + \Delta Debt} = \frac{t}{1-k_1} = \frac{0.2}{1-k_1}, \qquad (10)$$

Where:

t - The legally adopted income tax rate (in the RF, t = 20%);

k – The coefficient (share) of net reinvestments from the after-tax earnings;

A – The amortization,

Inv – The summed investments/disinvestments (in non-current assets and current capital) for a certain period;

$\Delta Debt$ – The newly attracted funds minus the returned loan funds for a certain period;

k_1 – The coefficient (share) of net reinvestments from before-tax operating income (summed investments in current and non-current capital – amortization and attracted loan balance/before-tax operating earnings).

Equations (10) were formulated by the author on the basis of the condition that after-tax cash flows equate to before-tax cash flows corrected for the levied (“effective”) income tax rate related to the entire cash flow masses. In other words, t_{efE} is a tax rate referred to the whole balance of free cash flows to equity - i.e., this is the share of paid income tax from the balance of before-tax free cash flows to equity, which is calculated on the level of the issuer:

$$FCFE_{BT}(1-t_{efE}) = (EBT + D - Inv +/- New\ Debt\ issue)(1-t_{efE}) = FCFE \qquad (11)$$

Where:

$FCFE_{BT}$ – The balance of the free cash flows to the equity issuer before income tax paid;

$FCFE$ – The balance of the free cash flows to the equity issuer after income tax paid;

Regarding equation (11), it is possible to give one more technique for calculating the effective tax rate:

$$t_{efE} = 1 - \frac{FCFE}{FCFE_{BT}}, \qquad (12)$$

Let’s give examples.

Example 1. Let’s suppose that there is a reason to expect that in the nearest year the operating earnings before taxes (EBT) = 100, the amortization = 30, the amount of the summed investments = 40, the amount of loan funds will be unchanged. The income tax rate equals 20%. The before-tax discount rate, which is calculated by the market extraction method, amounts to 30%. In the future, the cash flows are supposed to grow annually by 3%, and the income tax rate will stay unchanged. Let's find the cost of the equity capital, using two approaches, i.e., those on the before-tax and after-tax bases.

The cost of the equity capital on the before-tax basis is calculated as:

$$V_{BT} = \frac{EBT + A - Inv}{r_{BT} - g} = \frac{100 + 30 - 40}{0.3 - 0.03} = 333.$$

To find the cost on the after-tax basis, it is necessary to calculate the values of the after-tax cash flow and before-tax discount rate. The after-tax cash flow is found as:

$$\text{FCFE} = \text{EBT}(1\text{-t}) + \text{A} - \text{Inv} = 80 + 30 - 40 = 70$$

In accordance with (8), the after-tax discount flow is found as:

$$r = g + (r_{BT} - g)(1 - t_{efE}) = 0.03 + (0.3 - 0.03)(1 - 0.222) = 0.24,$$

Where, in accordance to (10), t_{efE} equates to:

$$t_{efE} = \frac{EBT \times t}{EBT + A - Inv + \Delta Debt} = \frac{100 \times 0.2}{100 + 30 - 40 + 0} = \frac{20}{90} = 0.222,$$

With regard to the obtained parameters, the cost of the equity capital on the after-tax basis, in accordance with (5), is calculated as:

$$V = \frac{FCFE}{r - g} = \frac{70}{0.24 - 0.03} = 333.$$

The results of this example clearly show the identity of the calculations done on the before-tax and after-tax bases, using the capitalized cash flow model.

Let's give an example with the discounted cash flow model. Previously, it is to be mentioned that equations (6) ÷ (9) were developed from supposing the existence of the conditions for realizing the model of capitalization.

Example 2. Let's suppose that, as in the preceding example, the before-tax discount rate, which is found by the market extraction method, is expected to amount to 30%; moreover, the forecast of cash flows to equity looks as:

Table 1. Forecast of key cash flow components

Year/Component	**1**	**2**	**3**
EBT	100	115	120
Tax (t = 0,2)	20	23	24
EBT-Tax= EBT(1-t))	80	92	96
Amortization	30	30.3	30.7
Inv	40	44	45
Newly attracted loan balances	+5	-10	0
Before-tax free cash flow to equity (FCFEBT)	95	91.3	105.7
After-tax free cash flow to equity (FCFE)	75	68.3	81.7

In the future, the cash flows are supposed to grow annually by 3%, and the income tax rate will be unchanged.

Let's find the cost of the equity capital, using two approaches – those on the before-tax and after-tax bases.

The cost of the equity capital on the before-tax basis is calculated as:

$$V_{BT}=\frac{FCFE_{BT1}}{1+r_{BT}}+\frac{FCFE_{BT2}}{(1+r_{BT})^2}+\frac{FCFE_{BT3}}{(1+r_{BT})^3}+\frac{FCFE_{BT3}(1+g)}{(r_{BT}-g)(1+r_{BT})^3}=$$

$$\frac{95}{1.3}+\frac{91.3}{1.3^2}+\frac{105.7}{1.3^3}+\frac{105.7\times 1.03}{(0.3-0.03)\times 1.3^3}=359.$$

To find the cost of the equity capital on the after-tax basis, let's calculate the value of the discount rate on the after-tax basis; to do this, let's primarily estimate the values of the effective tax rates in accordance to (12) (see Table 2).

Table 2. Calculation of effective tax rates

Year/Component	1	2	3
Before-tax free cash flow to equity ($FCFE_{BT}$)	95	91.3	105.7
After-tax cash flow to equity (FCFE)	75	68.3	81.7
Effective tax rate (t_{efE})	0.211	0.252	0.227

In accordance with (6), let's calculate the values of the after-tax discount rates:

$$r_1=r_{BT}\times(1-t_{efE1})=0.3\times(1-0.211)=0.237\,,$$

$$r_2=r_{BT}\times(1-t_{efE2})=0.3\times(1-0.252)=0.224\,,$$

$$r_3=r_{BT}\times(1-t_{efE3})=0.3\times(1-0.227)=0.232\,,$$

$$r_{postforecast}=g+(r_{BT}-g)(1-t_{ef.postforecast})=0.03+(0.3-0.03)(1-0.227)=$$
$$=0.239\,,$$

Now we have all the necessary data for calculating the cost of the equity capital on the after-tax basis:

$$V=\frac{FCFE_1}{1+r_1}+\frac{FCFE_2}{(1+r_1)(1+r_2)}+\frac{FCFE_3}{(1+r_1)(1+r_2)(1+r_3)}+$$
$$+\frac{FCFE_3(1+g)}{(r_{postforecast}-g)(1+r_1)(1+r_2)(1+r_3)}=$$
$$\frac{75}{1.237}+\frac{68.3}{1.237\times 1.224}+\frac{81.7}{1.237\times 1.224\times 1.232}+$$
$$+\frac{81.7\times 1.03}{(0.239-0.03)\times 1.237\times 1.224\times 1.232}=365.$$

As it is obvious from the obtained figures, both valuation methods – those on the after-tax and before-tax bases – give very close results (359 and 365). The observed difference in the results can be explained. The techniques for correlating the after-tax and before-tax discount rates were introduced on the basis of the models of capitalization. That is why example 1 (with the model of capitalization) shows the results, which were calculated on the after-tax and before-tax bases, being totally the same; as soon as in example 2 (with the discounting model), the results aren't similar. The greater the difference in the results will become under the discounting model, the longer the forecasting period will last and the more significant the changes in cash flows will be expected in this period.

Let's discuss the tax aspects while finding the historical (retrospective) return on equity. This is useful as on this basis analysts and estimators set a premium for the risk of investments in equity capital. Returns on investments in equities are a combination of both price gains and dividend yields. Schematically, this relation can be showed as:

$$R_e = R_{price} + R_{div}, \tag{13}$$

Where:

R_e – The summed (aggregate) returns on equities;

R_{price} – The stock price gains,

R_{div} – The dividend returns on equities.

The aforementioned equation is widely known. Nevertheless, one can rarely meet the reference to how the tax factor is taken into account while valuating the total return on investments in equities. The main feature of calculating the total return on investments in equities on the after-tax and before-tax bases is associated with the phenomena, when price gains are often calculated on the before-tax basis while dividend yields are calculated on the after-tax basis (on the level of the equity issuer because of the dividend being paid after paying income tax). According to these notes, the following techniques are proposed to calculate returns on investments in equities on the before-tax and after-tax bases:

$$r_{BT} = r_{priceBT} + \frac{r_{div}}{1-t} \tag{14}$$

$$r = r_{price} + r_{div}, \tag{15}$$

Where:

r_{BT} - The aggregate before-tax returns on equities;

$r_{курсBT}$ - The before-tax stock price gains;

r – The aggregate after-tax returns on equities;

r_{price} - The after-tax stock price gains;

r_{div} - The dividend yields on equities;

t - The income tax rate (in general, effective income tax rate).

Let's propose the ways to calculate premiums for the risk of investments in equities with due account for the tax nuances discussed hereinbefore:

$$pr_{BT} = r_{BT} - r_f, \tag{16}$$

$$pr_{BT} = r_{BT} - r_f, \tag{17}$$

$$pr = r - r_f(1-t), \tag{18}$$

$$pr_{priceBT} = r_{priceBT} - r_f, \tag{19}$$

Where:

pr_{BT} – The before-tax premium for the risk of investments in equities;

pr – The after-tax premium for the risk of investments in equities;

pr_{price} – The after-tax premium for the risk of investments in equities with regard only to price gains;

$pr_{beforeBT}$ – The before-tax premium for the risk of investments in equities with regard only to price gains;

r_f – The before-tax risk-free rate of yield.

Here a sacramental question may occur from equations (16) - (19) whether it appears that a risk-free rate isn't a whole unit, being divided into before-tax and after-tax rates. This statement is true.

Let's see the example.

Example 3. Let's consider the return on a one-year FinMin bond as risk-free return, and its cost amounts to 100 RUB while the weighted average price for this bond in the auction equals 90 RUB. Let's suppose that the return on the investments in bonds is taxed at the rate of 20% (for legal entities or at that of 13% for natural persons). Then, under this conditions the before-tax returns for a legal entity (the discount rate) will amount to 10/90 = 0.111 = 11.1%, and the after-tax returns (the discount rate applied to after-tax income) will be equal to 10 × (1 – 0.2)/90 = 0.08888 = 8.89%.

If investors orient to the before-tax returns of 11.1%, the calculation bid price per bond will amount to 100 RUB (the expected before-tax income)/1.11 = 90 RUB. If investors orient to the after-tax returns, they should discount the after-tax income as 98 RUB/1.089 = 90 RUB. Both calculation ways, herein, on the after-tax and before tax bases result in the same figures. It is necessary to point out that for the conditions of this example there are two more potential calculation techniques leading to incorrect results: 98 RUB (after-tax income)/1.11 (before-tax income) = 88.21 RUB, and 100 RUB (before-tax income)/1.089 (after-tax income) = 91.83 RUB (the first invalid technique results in the lower figure, the second one shows the higher result).

Let's view the next example.

Example 4. Let's suppose that in the process of estimating the discount rate, the estimator finds that the rate of returns on government bonds amounts to 10%, the Beta coefficient equals 1 (let's suppose that the company has no loan funds; although in absence of this supposition, the logic of further calculations will be the same), and the risk premium amounts to 8%. The appropriate discount rate should be found (this task is typical and regular for almost every business valuation report).

As it is clear from the preceding example, to solve this task, one should determine the format for the input and output data. Let's suppose, as it often happens, that the estimator is going to discount the after-tax cash flow to equity. Accordingly, the *after-tax* rate of discounted equity capital is to be calculated. Let's start solving the task with calculating the effective tax rate. Let's suppose that with regard to the profile of the expected cash flows after the calculations conducted in accordance with (10) and (12), the appropriate value for effective tax rate amounts to 25%. Taking the findings into account, let's proceed with analyzing the value of risk-free returns; in this case, the before-tax return amount to 10% and the after-tax return equates to 10% × (1 – 0.25) = 7.5%. Now let's calculate the risk premium. Here one needs to understand what it deals with. There can be various variants including the premium on either the *before-tax* or *after-tax* basis, with regard to or regardless of dividend yields (depending on the way this premium has been calculated). Let's suppose that this premium is calculated on the before-tax basis and doesn't account for dividend yields. Then, after withholding the income tax, this premium will be equal to 8% × (1 – 0.25) = 6%. Finally, the value of dividend yields is to be found. Let's suppose that the analysis resulted in the average dividend return amounting to 2.5%. Hence, considering all the preceding findings, the final value of the after-tax discount rate will equate to 7.5% + 6% + 2.5% = 16%.

And what if we needed to find a before-tax discount rate? It would be calculated as: 10% + 8% + 2.5%/(1 – 0.25) = 21.33% (this rate of dividend yields (2.5%) is transformed into the before-tax format).

Let's check the obtained results: 21.33 × (1 – 0.25) = 16%. As it is obvious, the valuation results are the same, which would be impossible, using the traditional ways of valuating alternative yield rates.

2. Models of discounted/capitalized debt-free cash flows

Let's continue with viewing tax effects in the models of discounted debt-free flows, and let's look at the formation and calculation of cash flows for a certain period *on the level of investors:*

$$FCFF = EBIT(1-t) + D - Inv = \tag{20}$$

$$= EBI \times (1-k) = EBIT(1-t)(1-k), \tag{21}$$

Where:

FCFF – The balance of free cash flows to the firm; i.e., the balance of free cash flows to shareholders and to debtors giving loans at interest after income tax withheld and before personal taxes paid on shareholder's dividends.

EBIT – The earnings before interest and income tax payments;

D – The depreciation;

Inv – The summed investments and disinvestments in non-current assets and current capital for a certain period;

EBI – The earnings before interest payment;

k – The share of net reinvestments from earnings before interest payment;

t - The income tax rate.

Applying the capitalization models, it is obvious that the following equations should be satisfied:

[illegible] (22)

Where:

V – The value of income-bearing item subject to valuation;

FCFF – The after-tax free cash flow to the firm on the level of investors, expected in the nearest period;

$FCFF_{BT}$ – The before-tax cash flow to the firm, expected in the nearest period;

t_{ef} – The effective income tax rate (see the information hereinafter),

WACC – The after-tax rate of discounted cash flows to investors (weighted average after-tax alternative earnings/weighted average after-tax capital costs);

$WACC_{BT}$ – The before-tax rate of discounted cash flows (weighted average alternative before-tax earnings/ weighted average before-tax capital costs);

g – The expected rate of growth in cash flows.

From (22), the relation between WACC and WACC_{BT} becomes obvious:

$$WACC = g + (WACC_{BT} - g)(1 - t_{ef}) = WACC_{BT}(1 - t_{ef}) + gt_{ef}\,, \tag{23}$$

$$WACC_{BT} = g + \frac{WACC - g}{1 - t_{ef}}\,, \tag{24}$$

The aforementioned parameter t_{ef} - the effective income tax rate – is proposed to calculate as:

$$t_{ef} = \frac{t}{1-(1-t)k_2} = \frac{0.2}{1-0.8k_2} = \frac{EBIT \times t}{EBIT + A - Inv}\,, \tag{25}$$

Where:

t - The legally adopted income tax rate (in the RF, t = 20%);

A – The amortization;

Inv – The summed investments/disinvestments (into non-current assets and current capital) for a certain period;

k_2 – The coefficient (share) of net reinvestments from income before the interest is deducted (summed investments in current and non-current capital - amortization/income before deduction of interests).

(The development of the formula is beyond the scope of this paper).

Let's give an explanation: t_{ef} is the tax rate related to entire free cash flows; i.e., this is the share of paid income tax from before-tax free cash flows to investors:

$$FCFF_{BT}(1 - t_{ef}) = (EBIT + D - Inv)(1 - t_{ef}) = FCFF, \qquad (26)$$

Where:

FCFF$_{BT}$ – the balance of free cash flows to the firm before income tax payment;

FCFF – The balance of after-tax free cash flows to the firm.

Considering equation (26), one more technique for calculating the effective tax rate is proposed:

$$t_{ef} = 1 - \frac{FCFF}{FCFF_{BT}}. \qquad (27)$$

Let's give an example.

Example 5. Let's suppose that in the nearest period there is a reason to expect that the operating earnings before interest and tax payment (EBIT) will amount to 112, the amortization will be equal to 30 and the summed investments will be 40. The income tax rate is equal to 20%. The discount rate, which is associated with the weighted average before-tax costs of capital calculated with the market extraction method, amounts to 20%. In the future, the cash flows are supposed to grow annually at 3%, and the income tax rate is expected to be unchanged. Let's find the cost of the invested capital, using two approaches – those on the after-tax and before-tax bases.

The cost of the invested capital on the before-tax basis is calculated as:

$$V_{BT} = \frac{EBIT + A - Inv}{WACC_{BT} - g} = \frac{112 + 30 - 40}{0.2 - 0.03} = 600.$$

To evaluate the invested capital on the after-tax basis, the value of the after-tax cash flow and that of the after-tax discount rate are to be found:

The after-tax cash flow is calculated as:

$$FCFF = EBIT(1-t) + A - Inv = 89.6 + 30 - 40 = 79.6.$$

In accordance with (4.110), the after-tax discount rate equates to:

$$WACC = g + (WACC_{BT} - g)(1 - t_{ef}) = 0.03 + (0.2 - 0.03)(1 - 0.22) = 0.1626,$$

Where, in accordance with (25) and (27), t_{ef} amounts to:

$$t_{ef} = \frac{EBIT \times t}{EBIT + A - Inv} = \frac{112 \times 0.2}{112 + 30 - 40} = 1 - \frac{FCFF}{FCFF_{BT}} = 1 - \frac{79.6}{102} = 0.22.$$

According to (22) and with regard to the obtained parameters, the cost of equity capital on the after-tax basis is equal to:

$$V = \frac{FCFF}{WACC - g} = \frac{79.6}{0.1626 - 0.03} = 600.$$

The findings obtained in the example clearly show the identity of the calculations conducted on the after-tax and before-tax bases.

Example 6. As in the preceding example, the before-tax discount rate ($WACC_{BT}$), which is found with the market extraction method, is expected to be 20%; in the future, the cash flows are supposed to grow annually at 3%, and the income tax rate will stay unchanged. The forecast of cash flows to equty is given in the Table 3.

Table 3. Cash flow forecast

Year/Component	1	2	3
EBIT	112	127	132
Interest (paid interest)	12	12	12
EBT (= EBIT – Interest)	100	115	120
Tax (= t×EBT)	20	23	24
EBIT(1-t)	89.6	101.6	105.6
Amortization	30	30.3	30.7
Inv	40	44	45
Before-tax cash flow to the firm ($FCFF_{BT}$)	99.6	113.3	117.7
After-tax cash flow to the firm (FCFF)	79.6	90.3	93.7

Let’s find the cost of the equity capital, using two approaches – those on the after-tax and before-tax bases.

The cost of the equity capital on the after-tax and before-tax bases is calculated as:

$$V_{BT} = \frac{FCFF_{BT1}}{1+WACC_{BT}} + \frac{FCFF_{BT2}}{(1+WACC_{BT})^2} + \frac{FCFF_{BT3}}{(1+WACC_{BT})^3} + \frac{FCFF_{BT3}(1+g)}{(WACC_{BT}-g)(1+WACC_{BT})^3} =$$

$$\frac{99.6}{1.2} + \frac{113.3}{1.2^2} + \frac{117.7}{1.2^3} + \frac{117.7\times 1.03}{(0.2-0.03)\times 1.2^3} = 642.$$

To calculate the cost of the equity capital on the after-tax basis, in accordance with (27), let’s find the effective tax rates (see table 4).

Table 4. Calculation of effective tax rates

Year/Component	1	2	3
Before-tax cash flow to the firm ($FCFF_{BT}$)	99.6	113.3	117.7
After-tax cash flow to the firm (FCFF)	79.6	90.3	93.7
Effective tax rate ($t_{эф}$)	0.201	0.203	0.204

In accordance with (23), let’s calculate the values of the after-tax discount rates:

$$WACC_1 = WACC_{BT} \times (1-t_{ef1}) = 0.2 \times (1-0.201) = 0.1598,$$

$$WACC_2 = WACC_{BT} \times (1-t_{ef2}) = 0.2 \times (1-0.203) = 0.1594,$$

$$WACC_3 = WACC_{BT} \times (1-t_{ef3}) = 0.2 \times (1-0.204) = 0.1592,$$

$$WACC_{postforecast} = g + (WACC_{BT} - g)(1-t_{ef.postforecast}) =$$
$$0.03 + (0.2-0.03)(1-0.204) = 0.1653.$$

(in this calculation, for each forecast we supposed that the after-tax rate was equal to the before-tax rate of a given year corrected for the effective income tax rate; this slightly differs from equation (23) developed for the conditions of continuous infinite growth)

Now, let’s calculate the costs of the equity capital on the after-tax basis:

$$V = \frac{FCFF_1}{1+WACC_1} + \frac{FCFF_2}{(1+WACC_1)(1+WACC_2)} + \frac{FCFF_3}{(1+WACC_1)(1+WACC_2)(1+WACC_3)} +$$
$$+ \frac{FCFF_3(1+g)}{(WACC_{postforecast} - g)(1+WACC_1)(1+WACC_2)(1+WACC_3)} =$$
$$\frac{79.6}{1.1598} + \frac{90.3}{1.1598 \times 1.1594} + \frac{93.7}{1.1598 \times 1.1594 \times 1.1592} +$$
$$+ \frac{93.7 \times 1.03}{(0.1653 - 0.03) \times 1.1598 \times 1.1594 \times 1.1592} = 654.$$

As it is obvious from the obtained figures, both valuation ways – those on the after-tax and before-tax bases – give close results (642 and 654).

Let's view the details of the weighted average before-tax and after –tax cost of the capital ($WACC_{BT}$) and ($WACC$):

$$WACC_{BT} = w_e r_{eBT} + w_d r_D, \tag{28}$$

$$WACC = w_e r_e + w_d r_D (1 - t), \tag{29}$$

Where:
w_e – The share of the equity capital in the invested capital;
w_d – The share of the loan funds in the invested capital;
r_{eBT} – The before-tax cost of the equity capital;
r_D – The before-tax cost of the loan funds;
r_e – The after tax cost of the equity capital;
t – The (effective) rate of income tax.

Doing such investment calculations, a question often occurs which weighted average cost of capital (that on either the after-tax or before-tax basis) is calculated by estimators and investment analysts if it is necessary to apply the discounted debt-free cash flow model. The author of this paper gives the unambiguous answer to this question – the type of the applied rate should correspond to the type of the cash flow. As after-tax flows are mostly used while the discounted debt-free cash flow models are applied, they are to be discounted at after-tax rates. Correspondingly, estimating the weighted average after-tax costs of capital requires that the after-tax costs of capital are to be applied as the parameter r_e.

3. Expected growth rate with regard to the tax factor

To finish viewing the consideration of taxation in valuating discount rates, let's return to equations (6)-(7) and (23)-(24) developed hereinabove:

$$r = g + (r_{BT} - g)(1 - t_{efE}) = r_{BT}(1 - t_{efE}) + g t_{efE},$$

$$r_{BT} = g + \frac{r - g}{1 - t_{efE}} = \frac{r - g t_{efE}}{1 - t_{efE}},$$

$$WACC = g + (WACC_{BT} - g)(1 - t_{ef}) = WACC_{BT}(1 - t_{ef}) + g t_{ef},$$

$$WACC_{BT} = g + \frac{WACC - g}{1 - t_{ef}}.$$

As it is obvious from the aforementioned equations, to satisfy the equality of the final costs obtained on the basis of after-tax and before-tax calculation schemes [the conditions of (5) and (22)] at the grow rate (g) distinct from zero, the after-tax rate is to exceed (in case of a positive g) the before-tax rate, which is multiplied by the tax multiplier (1 - t_{ef}), by the value gt_{ef}. Another way to satisfy the equality of "before-tax" and "after-tax" costs is to adjust the expected rate of growth in income cash flows, applied for the calculation. For example, if the value of before-tax rate (r_{BT}) is known and the calculation is to be conducted, using the tax calculation base, it can be developed as:

$$V = \frac{CF_{AT}}{r_{BT}(1 - t_{ef}) + gt_{ef} - g} = \frac{CF_{AT}}{r_{BT}(1 - t_{ef}) - g(1 - t_{ef})}. \tag{30}$$

It is necessary to point out one more peculiarity – if the "addition" gt_{ef} is not taken into account (i.e., if a simplified relation between after-tax and before-tax rates is applied: $r_{AT} = r_{BT}(1 - t_{ef})$), there are the rates (a before-tax rate and an after-tax one), at which the results of calculation on the before-tax and after-tax bases are the same. For example, if g = 5%, t_{ef} = 20%, i.e. the value of before-tax flows for the next year being at the level of 105 c.u., the values of these "equilibrium" rates equate to r_{BT} = 12.03%, r_{AT} = 9.62%; at these values, the cost values on the after-tax and before-tax bases amount to 1198. In any deviation from these "equilibrium" rates, the difference in the results of the after-tax and before-tax estimations grows. Moreover, it grows at higher rates when the rates are decreased lower than the "equilibrium level"; and while the rates are increased higher than this level, the difference in estimations asymptomatically tends to the critical level.

As the supposition on the equality of the rates of growth in after-tax and before-tax income cash flows can be practically groundless (e.g., in case differential tax rates at various income levels are applied), equations (5) and (22) can be specified as:

$$V_E = \frac{FCFE_s}{r_s - g_{AT}} = \frac{FCFE(1 - t_{def})}{r_s - g_{AT}} = \frac{FCFE_{BT}(1 - t_{efE})}{r - g_{BT}} = \frac{FCFE}{r - g_{AT}} = \frac{FCFE_{BT}}{r_{BT} - g_{BT}}, \tag{31}$$

$$V = \frac{FCFF}{WACC - g_{AT}} = \frac{FCFF_{BT}(1 - t_{ef})}{WACC - g_{AT}} = \frac{FCFF_{BT}}{WACC_{BT} - g_{BT}}, \tag{32}$$

Where:
g_{AT} – The rate of growth in after-tax cash flows;
g_{BT} – The rate of growth in before-tax cash flows.
From (31) and (32), let's develop the specified equations connecting the after-tax and before-tax rates:

$$r = g_{AT} + (r_{BT} - g_{BT})(1 - t_{efE}), \tag{33}$$

$$r_{BT} = g_{BT} + \frac{r - g_{AT}}{1 - t_{efE}}, \tag{34}$$

$$WACC = g_{AT} + (WACC_{BT} - g_{BT})(1 - t_{ef}), \quad (35)$$

$$WACC_{BT} = g_{BT} + \frac{WACC - g_{AT}}{1 - t_{ef}}. \quad (36)$$

4. Summary of peculiarities of taking into account taxation in discounting models

The objective of the conducted research was to emphasize the prevention of a certain type of cash flow incorrectly mixed with the wrong discount or capitalization rates. The techniques demonstrated in this section and the examples of dual calculation of the costs of items subject to valuation should orient to the two elements: firstly, the principle of the relationship between the numerator (cash flows) and denominator (interest rate for adjustment) in the discounting models; secondly, the criterion of correct calculation, which is contextually associated with the equality of the results of the two calculation methods obtained on the after-tax and before-tax bases.

One can protest against such a detailed view of calculating the rate of return and reason his opinion that all these calculations are very complicated (as different market entities may have (and often do have) different tax rates and nobody knows average market tax rate); hence, market participants can’t account for these components even if they want to. The author of the conducted research can give two solutions of the issue. Firstly, the international valuation standards stipulate that estimators are to determine market borders in the process of valuation. Hence, while determining the borders, the market is allowed to be narrowed to the segment of legal entities-residents (or any other segment). Secondly, both now and always, market participants have mostly been oriented to the existing current mainstream level (i.e., undoubtedly, they have been lagging behind the leading scientific idea). Nevertheless, in full accordance with the proverb “the world never stays the same”, the scientific developments have influenced investors’ views (e.g., the Fisher formulas, the Black-Scholes models, the MM theorem, and the models of CAPM, APT, option pricing).

Thus, finishing the discussion on correctly taking into account the tax factor in the discounting models, we can make the following **conclusions**:

- While discounting, it is necessary to realize which discount rate is to be used. This rate is to precisely correspond to the type of discounted cash flow;
- For expected long-term (tending to infinity) periods of generated cash flows, there exist a correlation between the discount rate applied to discounting after-tax and before-tax cash flows;
- For a limited period of generated cash flows, there isn’t a (simple analytical) correlation between the discount rates found on the before-tax and after-tax bases;
- As after-tax cash flows are mostly subject to discounting, after-tax alternative return rates should be used as discount rates;
- In the present time, most calculations conducted by estimators, stock and investment analysts face the error of method, i.e., they apply before-tax discount rates to after-tax cash flows because of taking both before-tax risk-free returns and before-tax premiums for the risk of investments in equity capital. This error of method systematically provokes low valuation results obtained with the income approach.

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